ARTICLE XIII.

Practical Rule for Calculating, from the Elements in the Nautical Almanac, the Circumstances of an Eclipse of the Sun, for a Particular Place. By John Gummere, Teacher of Natural Philosophy and Mathematics in the Friends' School at Haverford, Pennsylvania. Read March 6th, 1835.

THE following rule, deduced from a known formula, gives, with little labour, the different circumstances of an eclipse of the sun, very nearly; the greatest error in time seldom exceeding half a minute. It also furnishes certain data that facilitate the exact calculation, when this is required. The multiplication of quantities by the sine, or cosine of an arc or angle, is performed by a Traverse Table, as in Henderson's method of Predicting Occultations, given in the fourth volume of the Memoirs of the Astronomical Society of London. The rule is adapted to the use of the traverse tables usually contained in treatises on Surveying. In these tables, the difference of latitude and the departure are given for every quarter of a degree, of course, from 0° to 90°; and but little error results, if the required quantity is taken in the column corresponding to the course which is nearest to the given angle, without correction for the difference between the two. It is, however, easy to estimate and apply the proportional part, corresponding to this difference; and it is better to do so. When the given angle exceeds 90°, it must be subtracted from 180°, and the remainder taken as the course.

In calculating the parallaxes, the products of ten times the distance of the given place from the earth's centre, by the cosine and sine of its reduced latitude, are used. These products being constant for a given place, serve, when once obtained, for all calculations of eclipses of the sun, or of occultations for that place. Let them be denoted, respectively, by X and Y. Then, to obtain them, add respectively, to the logarithmic cosine and sine of the latitude of the place, the logarithms x and y, taken from Table I. of the annexed tables, with the latitude of the place as the argument, and reject 10 from the index of each sum. The results will be the logarithms of X and Y. These logarithms are used in the exact calculation of the parallaxes. The natural numbers corresponding to them, taken out to two decimal places, are the values of X and Y, that are used in the approximate calculation. These values are given in Table II. for each degree of latitude.

In the addition and subtraction of quantities, except those which are in time, the algebraic rules for performing these operations are to be observed. Wherever the rule directs the root of a quantity to be taken, it is the positive square root that is implied.

The quantities, denominated in the rule, parallaxes in right ascension and declination, are not strictly those quantities; they, however, differ but little from them, and are the quantities required in this method of calculation.

RULE.

1. Consider north declinations and north horary motions as +, and south ones as —. Find the difference of the sun's and moon's declinations, by subtracting the declination of the sun, as given in the elements, from that of the moon. In like manner find the difference of the horary motions of the sun and moon, in right ascension, the difference of their horary motions in declination, and the difference of their equatorial horizontal parallaxes.

2. Multiply the difference of the sun's and moon's declinations, re-

duced to seconds, by 10, and divide the product by the difference of the parallaxes, also reduced to seconds, extending the quotient to two decimal places, and denote it by q. Do the same with the difference of the horary motions in right ascension, denoting the quotient by P'; with the difference of the horary motions in declination, denoting the quotient by q'; and with the sun's semidiameter, denoting the quotient by r. With the moon's declination, as a course, and P' as a distance, enter the traverse table, and taking the corresponding difference of latitude, mark it $+$, and denote it by p'. Then will q, p', q' and r, respectively express the difference of the declinations, the difference of the horary motions in right ascension, the difference of the horary motions in declination, and the sun's semidiameter, in such parts as the difference of the parallaxes contains 10; the difference of the horary motions in right ascension being reduced to the parallel of declination passing through the moon's centre. Let p denote the difference of the sun's and moon's right ascensions, expressed in similar parts, and reduced to the same parallel. At the time of conjunction in right ascension $p = 0$.

3. Denote the Greenwich mean time of conjunction in right ascension by T. Find from the Nautical Almanac the corresponding equation of time, and apply it to T, so as to obtain the apparent time. To the apparent time apply the longitude of the given place from Greenwich, in time, by adding when the longitude is east, but subtracting when it is west, and convert the sum or remainder into degrees. If the result is less than 180°, it will be the hour angle at the time T, and will be $+$. If it exceeds 180°, subtract it from 360°, and the remainder will be the hour angle, and will be $-$. Denote the hour angle by H.

With the sun's declination as a course, and the value of Y for the given place as a distance, enter the traverse table, and take the corresponding difference of latitude, marking it $+$ when the latitude of the place is north, but $-$ when it is south, and denote it by b. Take also the departure, marking it with the same sign as the declination when the latitude is north, but with a contrary sign when it is south, and denote it by f.

4. The values of p', q', r, b, and f, may be regarded as constant

during the continuance of the eclipse. But the value $p = 0$, and the values of q and H, found as above, appertain only to the time T. To find them for another time T$'$, proceed thus. As 60 minutes : diff. of T and T$'$: : p' : p. If T$'$ is later than T, the value of p is $+$, but if earlier, it is $-$. Again, as 60 minutes : diff. of T and T$'$: : q' : a quantity with the same sign as q', which, added to the value of q, at the time T, when T$'$ is later than T, but subtracted from it when T$'$ is earlier, will give the required value of q. Also, as 60 minutes : diff. of T and T$'$: : 15° : a quantity, which added to the value of H, at the time T, when T$'$ is later than T, but subtracted when it is earlier, will give the required value of H.

5. With the value of H, at the time T, as a course, and the value of X, for the given place, as a distance, enter the traverse table, and take the corresponding departure, marking it with the same sign as H, and denoting it by u. Take also the difference of latitude, marking it $+$, when H is less than 90°, but $-$, when H exceeds 90°, and denote it by C. With the sun's declination as a course, and C as a distance, find the departure, marking with the same sign as C when the declination is $+$, but with a contrary sign when it is $-$, and denote it by c. Subtract c from b, and denote the result by v. Then will u and v be the parallaxes in right ascension and declination, at the time T.

Using Table IV., add together the squares of $(p - u)$ and $(q - v)$, denoting the root of the sum, which need not however be taken out, by M. Then will M denote the apparent distance of the centres of the sun and moon, at the time T.

6. Take a time T$'$, an hour earlier or later than T, according as the value of $(p - u)$ at the time T, is $+$ or $-$, and find for this time, by the preceding articles, the values of p, q, H, u, C, c, and v; and thence the square of the apparent distance of the centres, denoting the root by M$'$.

Subtract, respectively, the values of $(p - u)$ and $(q - v)$ at the earlier of the times T and T$'$, from their values at the later time, and denote the results by $(p' - u')$ and $(q' - v')$. Add together the squares of $(p' - u')$ and $(q' - v')$, and taking from the table the corresponding root, denote it by n. Then will n express the horary motion of the moon from the sun on the apparent relative orbit. To the square of n add the square of M, and from the sum subtract the square of M′, denoting the remainder by N^2. Multiply N^2 by 30, and divide the product by the square of n, extending the quotient to one decimal figure. This quotient will be an interval in minutes of time, which, being added to the time T, or subtracted from it, according as T′ is later or earlier than T, will give the time of greatest obscuration.

7. Taking now T′, to represent the time of greatest obscuration, find for this time the values of p, q, H, u, C, c, and v. Also, when taking c from the traverse table, take the corresponding difference of latitude, and marking it with the same sign as C, denote it by g. With $(f + g)$ as the argument, take from Table IX., to two figures, the correction of r. Subtract this correction from r, and denote the remainder by r'. To r' add 2.73, the moon's reduced semidiameter, and denote the sum by k. Now adding together the squares of $(p - u)$ and $(q - v)$, take the root of the sum, and denote it by m. Then will m express the least distance of the centres. Multiply $(k - m)$ by 6, and divide the product by r'. The quotient will express the *digits eclipsed;* on the northern limb if $(q - v)$ is +, but on the southern if it is —. If m is equal to, or greater than k, the eclipse will not be visible at the given place.

From the square of k subtract the square of m, and taking the root of the remainder, denote it by h. Then, as $n : h :: 60$ minutes : an interval of time, which being subtracted from the time of greatest obscuration, and added to it, will give approximate times of the beginning and end of the eclipse.

8. Taking T′ equal the approximate time of beginning, find as before, for this time, the values of p, q, H, u, C, c, g, v, r', and k. Also with the sun's declination as a course, and u as a distance, find the corresponding departure, marking it with the same sign as u, when the declination is +, but with a contrary sign when the declination is —, and denote it by E. Then with C and E, respectively as arguments, take the corresponding quantities from Table III., marking each with the same sign as its argument, and denote them by u' and v'. Then will u' and v' express the horary changes of the parallaxes in right ascension and declination, at the time T′.

From the square of k, subtract the square of m, and taking the root, denote it by h. Add together the squares of $(p - u)$ and $(q - v)$, and from the sum subtract the square of m. Take the root of the remainder, and denote it by h'. Add together the squares of $(p' - u')$ and $(q' - v')$, and taking the root of the sum, it will be the value of n, at the time T'. Then as n : diff. of h and h' :: 60 minutes : a correction, in minutes, which being added to T', or subtracted from it, according as h' is greater or less than h, will give the *corrected time of beginning*.

9. The corrected time of end is found in exactly the same manner, except that the correction is to be subtracted from T', the approximate time of end, when h' is greater than h, but added to it when h' is less than h.

10. From the values of $(p - u)$, $(q - v)$, u and v, at the approximate time of beginning, find, by means of their horary changes $(p' - u')$, $(q' - v')$, u' and v', their values at the corrected time of beginning. Then taking the values of $(p - u)$ and $(q - v)$, divide the less by the greater, extending the quotient to three decimal places, and marking it $+$ when the signs of $(p - u)$ and $(q - v)$ are alike, but $-$ when they are different. Then with the quotient as the argument, take the corresponding arc from the proper column of Table V., as denoted by the remarks at the head of the table. If $(p - u)$ is $+$, denote this arc by P, but if it is $-$, add 180° to the arc, and denote the sum by P. With the values of u and v, proceed in the same manner to find another arc, denoting it by Q, if u is $+$, but adding 180° to it if u is $-$, and denoting the sum by Q. Subtract P from Q, increasing the latter by 360° when it is less than the former, and denote the remainder by V. Then will V express the distance from the sun's vertex to the point of the disc at which the eclipse commences, measured on the circumference of the disc, from the vertex to the right hand.

11. The times of beginning, greatest obscuration, and end, found as above, are expressed in Greenwich mean time, and may be changed to mean time of the given place, by adding or subtracting the difference of meridians in time, according as the place is east or west from Greenwich.

Note 1. The calculation will be facilitated by having two small tables, containing the values of u and C, for each degree of the hour angle, and b and f for each degree of declination, calculated for the place, from the expressions $u = X \sin. H$, $C = X \cos. H$, $b = Y \cos. Decl.$, and $f = Y \sin. Decl.$ These tables will also be equally convenient in the calculation of occultations. Tables VI. and VII. contain those values, calculated for the latitude of Philadelphia.

2. If only a near approximation to the circumstances of the eclipse is required, the value of r may be used instead of r', and the values of h and n at the time of greatest obscuration may be taken, in finding the corrected times of beginning and end. Also in finding the point of the sun's disc at which the eclipse commences, the values of $(p - u)$, $(q - v)$, u and v, at the approximate time of beginning, may be used without correction; consequently, in this case f, g and E need not be found. The error thus produced in the time of beginning or end will seldom exceed a minute; and the error in the magnitude of the eclipse cannot amount to a tenth of a digit.

As an example, let it be required to calculate for Philadelphia, lat. 39° 57′ N. long., 5 h. 0 m. 44 sec. W., the circumstances of the eclipse of November 30th, 1834.

For Philadelphia $X = 7{\cdot}68$ and $Y = 6{\cdot}39$.

In the following calculation, the values of b, f, u and C are taken from Tables VI. and VII.; the same values will, however, be easily obtained from the traverse table, with perhaps occasionally a difference of a unit in the last decimal figure.

From the elements in the Nautical Almanac we obtain:

Greenwich mean time of conjunc. in R. A.,		Nov. 30d.	6h.	32·9m.	
Moon's declination, ...		— 20°	48′	13″	
Sun's declination, ...		— 21	41	05	
Sun's semidiameter, ...			16	15	
Diff. of sun's and moon's declinations,	= + 52′	52″			= + 3172″
Diff. of their hor. motions in R. A.,	= + 35	40			= + 2140
Diff. of their hor. motions in declin.,	= — 8	48			= — 528
Diff. of their eq. horizontal parallaxes,	= 60	14			= 3614

$$p = 0,\ q = +\frac{3172 \times 10}{3614} = +\ 8{\cdot}78,\ P' = +\frac{2140 \times 10}{3614} = +\ 5{\cdot}92\ ;$$

$$p' = +\ 5{\cdot}54,\ q' = -\frac{528 \times 10}{3614} = -\ 1{\cdot}46,\ r = \frac{975 \times 10}{3614} = 2{\cdot}70.$$

		h. m.	
T. . = . .		6 32·9	
Eq. of time, . .	+	11·1	$b = +\ 5{\cdot}93$
		6 44·0	
Long. W. . .	—	5 0·7	$f = -\ 2{\cdot}36$
		1 43·3	
H. . . .		+ 25°.8	

$u = +\ 3{\cdot}34$, $C = +\ 6{\cdot}91$, $c = -\ 2{\cdot}55$, $v = +\ 8{\cdot}48$.

$p - u = -\ 3{\cdot}34$ sq. 11·16
$q - v = +\ 0{\cdot}30$ sq. 0·09

M. . . . sq. 11·25

h. m.
$T' = 7$ 32·9

$p = +\ 5{\cdot}54$, $q = +\ 7{\cdot}32$, $H = +\ 40°{\cdot}8$
$u = +\ 5{\cdot}02$, $C = +\ 5{\cdot}81$, $c = -\ 2{\cdot}15$, $v = +\ 8{\cdot}08$

$p - u = +\ 0{\cdot}52$ sq. 0.27
$q - v = -\ 0{\cdot}76$ sq. 0·58

M′ . . . . sq. 0.85

$p' - u' = +\ 3{\cdot}86$ sq. 14·90
$q' - v' = +\ 1{\cdot}06$ sq. 1·12

$n = 4{\cdot}00$. sq. 16·02
M . . . . sq. 11·25

27·27
M′ . . . . sq. 0·85

N . . . . sq. 26·42

$$\frac{30\ N^2}{n^2} = \frac{26{\cdot}42 \times 30}{16{\cdot}02} = 49{\cdot}5 \text{ m.}$$

h. m. m. h. m.
6 32·9 + 49·5 = 7 22·4 = time of greatest obscuration.

h. m.
$T' = 7$ 22·4

$p = +\ 4{\cdot}57$, $q = 7{\cdot}58$, $H = +\ 38°{\cdot}2$
$u = +\ 4{\cdot}75$, $C = +\ 6{\cdot}03$, $c = -\ 2{\cdot}23$, $g = +\ 5{\cdot}54$, $v = +\ 8{\cdot}16$
$f + g = +\ 3{\cdot}18$, $r' = 2{\cdot}69$, $k = 5{\cdot}42$

$p - u = -\ 0{\cdot}18$ sq. 0·03
$q - v = -\ 0{\cdot}58$ sq. 0·34

$m = 0{\cdot}61$ sq. 0·37

$$\frac{6\,(k - m)}{r'} = \frac{4{\cdot}81 \times 6}{2{\cdot}69} = 10{\cdot}7 = \text{digits eclipsed on southern limb.}$$

$k = +\ 5{\cdot}42$ sq. $29{\cdot}38$
m . . sq. $0{\cdot}37$

$h = 5{\cdot}39$ sq. $29{\cdot}01$

m. h. m.
$4{\cdot}00 : 5{\cdot}39 :: 60 : 1\ 20{\cdot}8$

h. m. h. m. h. m.
7 22·4 — 1 20·8 = 6 1·6 = approx. time of beginning,
7 22·4 + 1 20·8 = 8 43·2 = approx. time of end.

h. m.
T′ = 6 1·6

$p = -\ 2{\cdot}89$, $q = +\ 9{\cdot}54$, $H = 18^\circ{\cdot}0$, $u = +\ 2{\cdot}37$, $C = +\ 7{\cdot}30$
$c = -\ 2{\cdot}70$, $g = +\ 6{\cdot}78$, $v = +\ 8{\cdot}63$, $f + g = 4{\cdot}42$, $r' = 2{\cdot}68$
$k = 5{\cdot}41$, $E = -\ 0{\cdot}88$, $u' = +\ 1{\cdot}91$, $v' = -\ 0{\cdot}24$,

$k =$	5·41,	sq. 29·27	$p - u =$	— 5·26	sq. 27·67
m	. .	sq. 0·37	$q - v =$	+ 0·91	sq. 0·83
$h =$	5·38	sq. 28·90			28·50
$h' =$	5·30		m	. . . .	sq. 0·37
Diff. =	0·08		h' . =	5·30	sq. 28·13

$p' - u' = +\ 3{\cdot}63$ sq. $13{\cdot}18$
$q' - v' = -\ 1{\cdot}22$ sq. $1{\cdot}49$

n . $= 3{\cdot}83$ sq. $14{\cdot}67$

m. m.
$3{\cdot}83 : 0{\cdot}08 :: 60 : 1{\cdot}3$

h. m. m. h. m.
6 1·6 — 1·3 = 6 0·3 = corrected time of beginning.

h. m.
T′ = 8 43·2

$p = +\ 12{\cdot}03$, $q = +\ 5{\cdot}61$, $H = 58^\circ{\cdot}4$, $u = +\ 6{\cdot}54$, $C = +\ 4{\cdot}02$
$c = -\ 1{\cdot}49$, $g = +\ 3{\cdot}74$, $v = +\ 7{\cdot}42$, $f + g = 1{\cdot}38$, $v' = 2{\cdot}69$
$k = 5{\cdot}42$, $E = -\ 2{\cdot}42$, $u' = +\ 1{\cdot}05$, $v' = -\ 0{\cdot}63$,

$k =$	5·42,	sq. 29·38,	$p - u =$	+ 5·49,	sq. 30·14
m	. .	sq. 0.37,	$q - v =$	— 1·81,	sq. 3·28
$h =$	5·39,	sq. 29·01,			33·42
			m	. . .	sq. 0·37
$p' - u' =$	+ 4·49,	sq. 20·16,	$h' =$	. 5·75,	sq. 33·05
$q' - v' =$	— 0·83,	sq. 0·69,	$h =$	. 5·39,	
$n =$	4·57,	sq. 20·85,	Diff. =	. 0·36.	

m. m.
$4{\cdot}57 : 0{\cdot}36 :: 60 : 4{\cdot}7$

h. m. m. h. m.
8 43·2 — 4·7 = 8 38·5 = corrected time of end.

h. m.
At. 6 0·3

$p - u = -5{\cdot}34,\quad q - v = +0{\cdot}94,\quad u = +2{\cdot}33,\quad v = +8{\cdot}64$

$$\frac{q-v}{p-u} = \frac{+0{\cdot}94}{-5{\cdot}34} = -{\cdot}176 \qquad P = 280^\circ{\cdot}0$$

$$\frac{u}{v} = \frac{+2{\cdot}33}{+8{\cdot}64} = +{\cdot}269 \qquad Q = 15^\circ{\cdot}1$$

$$V = Q - P = 95^\circ{\cdot}1$$

Changing the Greenwich mean times into Philadelphia mean times, we have,

	h.	m.
Beginning,	0	59·6
Greatest obscuration,	2	21·7
End,	3	37·8

The first part of the calculation, by note 2d to the rule, is the same as the preceding, except that *f* need not be found. The subsequent part, after the time of greatest obscuration is obtained, is as follows:—

h. m.
T′ = 7 22·4

$p = +4{\cdot}57,\quad q = +7{\cdot}58,\quad H = +38^\circ{\cdot}2$

$u = +4{\cdot}75,\quad C = +6{\cdot}03,\quad c = -2{\cdot}23,\quad v = +8{\cdot}16$

$p - u = -0{\cdot}18$ sq. 0·03

$q - v = -0{\cdot}58$ sq. 0·34

$m = 0.61$ sq. 0.37

$$\frac{6\,(k-m)}{r} = \frac{4{\cdot}82 + 6}{2{\cdot}70} = 10{\cdot}7 = \text{digits eclipsed.}$$

$k = 5{\cdot}43$ sq. 29·48

m sq. 0·37

$h = 5{\cdot}40$ sq. 29·11

m. h. m.
4·00 : 5·40 : : 60 : 1 21·0

h. m. h. m. h. m.
7 22·4 — 1 21·0 = 6 1·4 = approx. time of begin.
7 22·4 + 1 21·0 = 8 43·4 = approx. time of end.

h. m.
T′ = 6 1·4

$p = -2{\cdot}91,\quad q = +9{\cdot}55,\quad H = 17^\circ{\cdot}9$

$u = +2{\cdot}36,\quad C = +7{\cdot}30,\quad c = -2{\cdot}70,\quad v = +8{\cdot}63$

$p - u = - 5{\cdot}27$	sq. 27·77	
$q - v = + 0{\cdot}92$	sq. 0·85	
	28·62	
m	sq. 0·37	
$h' = 5{\cdot}31$	sq. 28·25	
$h = 5{\cdot}40$		
Diff. $= 0{\cdot}09$		

m. m.
4·00 : 0·09 : : 60 : 1·3

h. m. m. h. m.
6 1·4 — 1·3 = 6 0·1 = near approx. time of begin.

h. m.
T′ = 8 43·4

$p = + 12{\cdot}05$, $q = + 5{\cdot}60$, $H = 58^\circ{\cdot}4$
$u = + 6{\cdot}54$, $C = + 4{\cdot}02$, $c = - 1{\cdot}49$, $v = + 7{\cdot}42$

$p - u = + 5{\cdot}51$	sq. 30·36	
$q - v = - 1{\cdot}82$	sq. 3·31	
	33·67	
m	sq. 0·37	
$h' = 5{\cdot}77$	sq. 33·30	
$h = 5{\cdot}40$		
Diff. $= 0{\cdot}37$		

m. m.
4·00 : 0·37 : : 60 : 5·5

h. m. m. h. m.
8 43·4 — 5·5 = 8 37·9 = near approx. time of end.

h. m.
At 6 1·4

$$\frac{q - v}{p - u} = \frac{+ 0{\cdot}92}{- 5{\cdot}27} = - {\cdot}174, \quad P = 279^\circ{\cdot}8$$

$$\frac{u}{v} = + \frac{2{\cdot}36}{8{\cdot}63} = + {\cdot}273, \quad Q = 15^\circ{\cdot}2$$

$$V = Q - P = 95^\circ{\cdot}4$$

If it is required to find the times of beginning and end with greater precision than by the foregoing rule, let T′ represent the corrected Greenwich mean time of beginning, taken to the nearest minute, and find from the Nautical Almanac the corresponding sidereal time, expressing it in arc. To the sidereal time thus expressed apply the longitude of the place, also in arc, by adding, if the longitude is east, but subtracting if it is west, and denote the result by Z. Find also, for the time T′, the sun's right ascension in arc, denoting it by A; the sun's declination, denoting it by D; the moon's right ascension, in arc, denoting it by a; the moon's declination, denoting it by d; and the moon's equatorial horizontal parallax. Take the difference of the sun's and moon's parallaxes, and denote it by G. Also denote the sun's semidiameter by R. Then find the values of p, q, r, u, and v, to four decimal places, by the following formulas.

$$p = \frac{10 \sin. (a - A) \cos. d}{\sin. G}$$

$$q = \frac{10 \sin. (d - D)}{\sin. G} + \tfrac{1}{2} p \sin. D \sin. (a - A)$$

$$r = \frac{10 \text{ tang. } R \cos. (d - D) \cos. (a - A)}{\sin. G}$$

$$u = X \sin (Z - A)$$

$$v = Y \cos. D - X \sin. D \cos. (Z - A)$$

Find the value of g, for the time T, as directed in the preceding rule, and with the argument $(f + g)$ take the correction of r from Table IX., and subtracting it from r, obtain r'. Take the moon's semidiameter from Table VIII., with the equatorial parallax as the argument, and adding it to r', the sum will be the value of k. The square of m, and the value of n, at the approximate time of beginning, found in the preceding calculation, although extending only to two decimal places, will be sufficiently accurate for the present calculation.

Using a common table of squares, and proportioning for the last two figures of the roots, find the values of h and h', as directed in article 8 of the foregoing rule, and thence a second correction; which being applied to T′, as there directed, will give the true time of beginning.

A similar calculation for the corrected time of end, will give the true time of end.

The corrected time of beginning of the eclipse just calculated, has been found to be 6 h. 0·3 m. Take therefore T′ = 6 h. 0 m. The sidereal time corresponding to this time is 339° 7′ 22″.2, expressed in arc. Hence for Philadelphia, long. 75° 10′ 59″ W., we have, Z = 263° 56′ 23″.2, at the time T′. We also find A = 246° 21′ 7″.8, D =

— 21° 40′ 51″.5, a = 246° 1′ 33″.1, d = — 20° 43′ 8″.1, moon's parallax = 60′ 23″.3, and R = 16′ 14″.8.

Hence Z — A = 17° 35′ 15″, a — A = — 19′ 34″.7, d — D = + 57′ 43″.4, and G = 60′ 14″.6.

With these values we obtain, p = — 3·0398, q = + 9·5787, r = 2·6965, u = + 2·3195, and v = + 8·6398.

The value of g, for the time T′, is + 6·80, and consequently (f + g) = + 4·44. This gives 0·0210 for the correction of r. Hence r' = 2·6755. The moon's semidiameter taken from Table VIII is 2·7315; consequently k = 5·4070. Then,

k = 5·4070	sq. 29·2357	$p - u$ =	— 5·3593	sq. 28·7221
m	sq. 0·3700	$q - v$ =	0·9389	sq. 0·8815
h = 5·3727	sq. 28·8657			29·6036
h' = 5·4068		m		sq. 0·3700
Diff. = 0·0341		h' = 5·4068		sq. 29·2336

m. m. sec.
3·83 : 0·0341 : : 60 : 0·53 = 32

Hence the true time of beginning is 6 h. 0 m. 32 sec., in Greenwich mean time.

For the end take T′ = 8 h. 38 m. Then we shall find Z = 303° 32′ 52″.6, A = 246° 28′ 13″.3, D = — 21° 41′ 55″.3, a = 247° 42′ 41″.8, d = — 21° 7′ 4″·3, moon's parallax = 60′ 21″.6, and as before R = 16′ 14″.8. We also find g = 3·88, and consequently f + g = 1·52.

Hence p = + 11·5373, q = + 5·7414, r = 2·6975, r' = 2·6903, k = 5·4218, u = + 6·4435 and v = + 7·4782.

k = 5·4218	sq. 29·3959	$p - u$ =	+ 5·0938	sq. 25·9468
m	sq. 0·3700	$q - v$ =	— 1·7368	sq. 3·0165
h = 5·3876	sq. 29·0259			28·9633
h' = 5·3473		m		sq. 0·3700
Diff. = 0·0403		h' = 5·3473		sq. 28·5933

m. m. sec.
4·57 : 0·0403 : : 60 : 0·53 = 32

Hence the true time of end is 8 h. 38 m. 32 sec. in Greenwich mean time. The true times of beginning and end, expressed in Philadelphia mean time, will be

	h.	m.	sec.
Beginning,	0	59	48
End,	3	37	48

It thus appears that in the present example the time of beginning, as found in the foregoing rule, differs only 12 seconds from the true time, and that the time of end exactly corresponds with that obtained by the exact calculation.

In these calculations no allowance has been made for irradiation and inflexion. To make this allowance we must diminish k, by subtracting from it the quotient of ten times the assumed value of these quantities, divided by the difference of the parallaxes in seconds. If we assume an irradiation and inflexion, amounting to $5''$, its effect in the present eclipse will be to make the time of beginning, at Philadelphia, 13 seconds later, and the time of end 11 seconds earlier than as above obtained. Thus we should have

	h.	m.	sec.
Beginning at	1	0	1
End at	3	37	37.

TABLE I.

Logarithms x and y.
Arg. *Latitude of Place.*

Arg.	Log. *x*.	Log. *y*.
0°	1·00000	0·99718
2	1·00000	0·99719
4	1·00001	0·99719
6	1·00002	0·99720
8	1·00003	0·99721
10	1·00004	0·99723
12	1·00006	0·99725
14	1·00008	0·99727
16	1·00011	0·99729
18	1·00013	0·99732
20	1·00016	0·99735
22	1·00020	0·99738
24	1·00023	0·99742
26	1·00027	0·99745
28	1·00031	0·99749
30	1·00035	0·99754
32	1·00039	0·99758
34	1·00044	0·99762
36	1·00048	0·99767
38	1·00053	0·99772
40	1·00058	0·99777
42	1·00063	0·99781
44	1·00068	0·99786
46	1·00073	0·99791
48	1·00078	0·99796
50	1·00082	0·99801
52	1·00087	0·99806
54	1·00092	0·99810
56	1·00097	0·99815
58	1·00101	0·99820
60	1·00105	0·99824
62	1·00110	0·99828
64	1·00114	0·99832
66	1·00117	0·99836
68	1·00121	0·99839
70	1·00124	0·99843
72	1·00127	0·99846
74	1·00130	0·99848
76	1·00133	0·99851
78	1·00135	0·99853
80	1·00137	0·99855
82	1·00138	0·99856
84	1·00139	0·99858
86	1·00140	0·99859
88	1·00141	0·99859
90	1·00141	0·99859

Note.—In the calculation of the above Table, the earth's compression was assumed to be $\frac{1}{309}$.

TABLE II.

Values of X *and* Y *for each Degree of Latitude.*

Lat.	X.	Y.	Lat.	X.	Y.
0°	10·00	0·00	45°	7·08	7·04
1	10·00	0·17	46	6·96	7·16
2	9·99	0·35	47	6·83	7·28
3	9·99	0·52	48	6·70	7·40
4	9·98	0·69	49	6·57	7·51
5	9·96	0·87	50	6·44	7·63
6	9·95	1·04	51	6·31	7·74
7	9·93	1·21	52	6·17	7·84
8	9·90	1·38	53	6·03	7·95
9	9·88	1·55	54	5·89	8·06
10	9·85	1·73	55	5·75	8·16
11	9·82	1·90	56	5·60	8·25
12	9·78	2·07	57	5·46	8·35
13	9·74	2·24	58	5·31	8·45
14	9·70	2·40	59	5·16	8·54
15	9·66	2·57	60	5·01	8·63
16	9·62	2·74	61	4·86	8·71
17	9·57	2·91	62	4·71	8·79
18	9·51	3·07	63	4.55	8·87
19	9·46	3·24	64	4·40	8·95
20	9·40	3·40	65	4·24	9·03
21	9·34	3·56	66	4·08	9·10
22	9·28	3·72	67	3·92	9·17
23	9·21	3·88	68	3·76	9·24
24	9·14	4·04	69	3·59	9·30
25	9·07	4·20	70	3·43	9·36
26	8 99	4·36	71	3·27	9·42
27	8·92	4·51	72	3·10	9·48
28	8·84	4·67	73	2·93	9·53
29	8·75	4·82	74	2·76	9·58
30	8·67	4·97	75	2·60	9·63
31	8·58	5·12	76	2·43	9·67
32	8·49	5·27	77	2·26	9·71
33	8·39	5·42	78	2·09	9·75
34	8·30	5·56	79	1·91	9·78
35	8·20	5·70	80	1·74	9·81
36	8·10	5·85	81	1·57	9·84
37	8·00	5·99	82	1·40	9·87
38	7·89	6·12	83	1·22	9·89
39	7·78	6·26	84	1·05	9·91
40	7·67	6·39	85	0·87	9·93
41	7·56	6·53	86	0·70	9·94
42	7·44	6·66	87	0·53	9·95
43	7·32	6·79	88	0·35	9·96
44	7·20	6·91	89	0·18	9·97
45	7·08	7·04	90	0·00	9·97

TABLE III.

Values of u′ and v′.
Arg. C *for u′*.
Arg. E *for v′*.

Arg.	*u′* or *v′*	Arg.	*u′* or *v′*
0·0	0·00	5·0	1·31
0·1	0·03	5·1	1·34
0·2	0·05	5·2	1·36
0·3	0·08	5·3	1·39
0·4	0·10	5·4	1·41
0·5	0·13	5·5	1·44
0·6	0·15	5·6	1·47
0·7	0·18	5·7	1·49
0·8	0·21	5·8	1·52
0·9	0·24	5·9	1·54
1·0	0·26	6·0	1·57
1·1	0·29	6·1	1·60
1·2	0·31	6·2	1·62
1·3	0·34	6·3	1·65
1·4	0·37	6·4	1·68
1·5	0·39	6·5	1·70
1·6	0·42	6·6	1·73
1·7	0·45	6·7	1·75
1·8	0·47	6·8	1·78
1·9	0·50	6·9	1·81
2·0	0·52	7.0	1·83
2·1	0·55	7·1	1·86
2·2	0·58	7·2	1·88
2·3	0·60	7·3	1·91
2·4	0·63	7·4	1·94
2·5	0·65	7·5	1·96
2·6	0·68	7·6	1·99
2·7	0·71	7·7	2·02
2·8	0·73	7·8	2·04
2·9	0·76	7·9	2·07
3·0	0·79	8·0	2·09
3·1	0·81	8·1	2·12
3·2	0·84	8·2	2·15
3·3	0·86	8·3	2·17
3·4	0·89	8·4	2·20
3·5	0·92	8·5	2·23
3·6	0·94	8·6	2·25
3·7	0·97	8·7	2·28
3·8	0·99	8·8	2·30
3·9	1·02	8·9	2·33
4·0	1·05	9·0	2·36
4·1	1·07	9·1	2·38
4·2	1·10	9·2	2·41
4·3	1·13	9·3	2·43
4·4	1·15	9·4	2·46
4·5	1·18	9·5	2·49
4·6	1·20	9·6	2·51
4·7	1·23	9·7	2·54
4·8	1·26	9·8	2.57
4·9	1·28	9·9	2·59
5·0	1·31	10·0	2·62

TABLE IV.

Squares of Numbers to two Decimal Places.

Root.	Square.	Root.	Square.	Root.	Square.	Root.	Square.	Root.	Square.	Root.	Square.
0·00	0·00	0·60	0·36	1·20	1·44	1·80	3·24	2·40	5·76	3·00	9.00
0·01	0·00	0·61	0·37	1·21	1·46	1·81	3·28	2·41	5·81	3·01	9·06
0·02	0·00	0·62	0·38	1·22	1·49	1·82	3·31	2·42	5·86	3·02	9·12
0·03	0·00	0·63	0·40	1·23	1·51	1·83	3·35	2·43	5·90	3·03	9·18
0·04	0·00	0·64	0·41	1·24	1·54	1·84	3·39	2·44	5·95	3·04	9·24
0·05	0·00	0·65	0·42	1·25	1·56	1·85	3·42	2·45	6·00	3·05	9·30
0·06	0·00	0·66	0·44	1·26	1·59	1·86	3·46	2·46	6·05	3·06	9·36
0·07	0·00	0·67	0·45	1·27	1·61	1·87	3·50	2·47	6·10	3·07	9·42
0·08	0·01	0·68	0·46	1·28	1·64	1·88	3·53	2·48	6·15	3·08	9·49
0·09	0·01	0·69	0·48	1·29	1·66	1·89	3·57	2·49	6·20	3·09	9·55
0·10	0·01	0·70	0·49	1·30	1·69	1·90	3·61	2·50	6·25	3·10	9·61
0·11	0·01	0·71	0·50	1·31	1·72	1·91	3·65	2·51	6·30	3·11	9·67
0·12	0·01	0·72	0·52	1·32	1·74	1·92	3·69	2·52	6·35	3·12	9·73
0·13	0·02	0·73	0·53	1·33	1·77	1·93	3·72	2·53	6·40	3·13	9·80
0·14	0·02	0·74	0·55	1·34	1·80	1·94	3·76	2·54	6·45	3·14	9·86
0·15	0·02	0·75	0·56	1·35	1·82	1·95	3·80	2·55	6·50	3·15	9·92
0·16	0·03	0·76	0·58	1·36	1·85	1·96	3·84	2·56	6·55	3·16	9·99
0·17	0·03	0·77	0·59	1·37	1·88	1·97	3·88	2·57	6·60	3·17	10·05
0·18	0·03	0·78	0·61	1·38	1·90	1·98	3·92	2·58	6·66	3·18	10·11
0·19	0·04	0·79	0·62	1·39	1·93	1·99	3·96	2·59	6·71	3·19	10·18
0·20	0·04	0·80	0·64	1·40	1·96	2·00	4·00	2·60	6·76	3·20	10·24
0·21	0·04	0·81	0·66	1·41	1·99	2·01	4·04	2·61	6·81	3·21	10·30
0·22	0·05	0·82	0·67	1·42	2·02	2·02	4·08	2·62	6·86	3·22	10·37
0·23	0·05	0·83	0·69	1·43	2·04	2·03	4·12	2·63	6·92	3·23	10·43
0·24	0·06	0·84	0·71	1·44	2·07	2·04	4·16	2·64	6·97	3·24	10·50
0·25	0·06	0·85	0·72	1·45	2·10	2·05	4·20	2·65	7·02	3·25	10·56
0·26	0·07	0·86	0·74	1·46	2·13	2·06	4·24	2·66	7·08	3·26	10·63
0·27	0·07	0·87	0·76	1·47	2·16	2·07	4·28	2·67	7·13	3·27	10·69
0·28	0·08	0·88	0·77	1·48	2·19	2·08	4·33	2·68	7·18	3·28	10·76
0·29	0·08	0·89	0·79	1·49	2·22	2·09	4·37	2·69	7·24	3·29	10·82
0·30	0·09	0·90	0·81	1·50	2·25	2·10	4·41	2·70	7·29	3·30	10·89
0·31	0·10	0·91	0·83	1·51	2·28	2·11	4·45	2·71	7·34	3·31	10·96
0·32	0·10	0·92	0·85	1·52	2·31	2·12	4·49	2·72	7·40	3·32	11·02
0·33	0·11	0·93	0·86	1·53	2·34	2·13	4·54	2·73	7·45	3·33	11·09
0·34	0·12	0·94	0·88	1·54	2·37	2·14	4·58	2·74	7·51	3·34	11·16
0·35	0·12	0·95	0·90	1·55	2·40	2·15	4·62	2·75	7·56	3·35	11·22
0·36	0·13	0·96	0·92	1·56	2·43	2·16	4·67	2·76	7·62	3·36	11·29
0·37	0·14	0·97	0·94	1·57	2·46	2·17	4·71	2·77	7·67	3·37	11·36
0·38	0·14	0·98	0·96	1·58	2·50	2·18	4·75	2·78	7·73	3·38	11·42
0·39	0·15	0·99	0·98	1·59	2·53	2·19	4·80	2·79	7·78	3·39	11·49
0·40	0·16	1·00	1·00	1·60	2·56	2·20	4·84	2·80	7·84	3·40	11·56
0·41	0·17	1·01	1·02	1·61	2·59	2·21	4·88	2·81	7·90	3·41	11·63
0·42	0·18	1·02	1·04	1·62	2·62	2·22	4·93	2·82	7·95	3·42	11·70
0·43	0·18	1·03	1·06	1·63	2·66	2·23	4·97	2·83	8·01	3·43	11·76
0·44	0·19	1·04	1·08	1·64	2·69	2·24	5·02	2·84	8·07	3·44	11·83
0·45	0·20	1·05	1·10	1·65	2·72	2·25	5·06	2·85	8·12	3·45	11·90
0·46	0·21	1·06	1·12	1·66	2·76	2·26	5.11	2·86	8·18	3·46	11·97
0·47	0·22	1·07	1·14	1·67	2·79	2·27	5·15	2·87	8·24	3·47	12·04
0·48	0·23	1·08	1·17	1·68	2·82	2·28	5·20	2·88	8·29	3·48	12·11
0·49	0·24	1·09	1·19	1·69	2·86	2·29	5·24	2·89	8·35	3·49	12·18
0·50	0·25	1·10	1·21	1·70	2·89	2·30	5·29	2·90	8·41	3·50	12·25
0·51	0·26	1·11	1·23	1·71	2·92	2·31	5·34	2·91	8·47	3·51	12·32
0·52	0·27	1·12	1·25	1·72	2·96	2·32	5·38	2·92	8·53	3·52	12·39
0·53	0·28	1·13	1·28	1·73	2·99	2·33	5·43	2·93	8·58	3·53	12·46
0·54	0·29	1·14	1·30	1·74	3·03	2·34	5·48	2·94	8·64	3·54	12·53
0·55	0·30	1·15	1·32	1·75	3·06	2·35	5·52	2·95	8·70	3·55	12·60
0·56	0·31	1·16	1·35	1·76	3·10	2·36	5·57	2·96	8·76	3·56	12·67
0·57	0·32	1·17	1·37	1·77	3·13	2·37	5·62	2·97	8·82	3·57	12·74
0·58	0·34	1·18	1·39	1·78	3·17	2·38	5·66	2·98	8·88	3·58	12·82
0·59	0·35	1·19	1·42	1·79	3·20	2·39	5·71	2·99	8·94	3·59	12·89
0·60	0·36	1·20	1·44	1·80	3·24	2·40	5·76	3·00	9·00	3·60	12·96

TABLE IV. CONTINUED.

Squares of Numbers to two Decimal Places.

Root.	Square.	Root.	Square.	Root.	Square.	Root.	Square.	Root.	Square.
3·60	12·96	4·20	17·64	4·80	23·04	5·40	29·16	6·00	36·00
3·61	13·03	4·21	17·72	4·81	23·14	5·41	29·27	6·01	36·12
3·62	13·10	4·22	17·81	4·82	23·23	5·42	29·38	6·02	36·24
3·63	13·18	4·23	17·89	4·83	23·33	5·43	29·48	6·03	36·36
3·64	13·25	4·24	17·98	4·84	23·43	5·44	29·59	6·04	36·48
3·65	13·32	4·25	18·06	4·85	23·52	5·45	29·70	6·05	36·60
3.66	13·40	4·26	18·15	4·86	23·62	5·46	29·81	6·06	36·72
3·67	13·47	4·27	18·23	4·87	23·72	5 47	29·92	6 07	36 84
3·68	13·54	4·28	18·32	4·88	23·81	5·48	30·03	6·08	36·97
3·69	13·62	4·29	18·40	4·89	23·91	5·49	30·14	6·09	37·09
3·70	13·69	4·30	18·49	4·90	24·01	5·50	30·25	6·10	37·21
3·71	13·76	4·31	18·58	4·91	24·11	5·51	30·36	6·11	37·33
3·72	13·84	4·32	18·66	4·92	24·21	5·52	30·47	6·12	37·45
3·73	13·91	4·33	18·75	4·93	24·30	5·53	30·58	6 13	37·58
3·74	13·99	4·34	18·84	4·94	24·40	5·54	30·69	6·14	37·70
3·75	14·06	4·35	18·92	4·95	24·50	5·55	30·80	6-15	37·82
3·76	14·14	4·36	19·01	4·96	24·60	5·56	30·91	6·16	37·95
3·77	14·21	4·37	19·10	4·97	24·70	5·57	31·02	6·17	38·07
3·78	14·29	4·38	19·18	4·98	24·80	5·58	31·14	6·18	38·19
3·79	14·36	4·39	19·27	4·99	24·90	5·59	31·25	6·19	38·32
3·80	14·44	4·40	19·36	5·00	25·00	5·60	31·36	6·20	38·44
3·81	14·52	4·41	19.45	5·01	25·10	5·61	31·47	6·21	38·56
3·82	14·59	4·42	19·54	5·02	25·20	5·62	31·58	6·22	38·69
3·83	14·67	4·43	19·62	5·03	25·30	5·63	31·70	6·23	38·81
3·84	14·75	4·44	19·71	5·04	25·40	5·64	31·81	6·24	38·94
3·85	14·82	4·45	19·80	5·05	25·50	5·65	31·92	6·25	39·06
3·86	14.90	4·46	19·89	5·06	25·60	5·66	32·04	6·26	39·19
3·87	14·98	4·47	19·98	5·07	25·70	5·67	32·15	6·27	39·31
3.88	15·05	4·48	20·07	5·08	25·81	5·68	32·26	6·28	39·44
3·89	15·13	4·49	20·16	5·09	25·91	5.69	32·38	6·29	39·56
3·90	15·21	4·50	20·25	5·10	26·01	5·70	32·49	6·30	39·69
3·91	15·29	4·51	20·34	5·11	26·11	5·71	32·60	6·31	39·82
3·92	15·37	4·52	20·43	5·12	26·21	5·72	32·72	6·32	39·94
3·93	15·44	4·53	20·52	5·13	26·32	5·73	32·83	6·33	40·07
3·94	15·52	4·54	20·61	5·14	26·42	5·74	32·95	6·34	40·20
3·95	15·60	4·55	20·70	5·15	26·52	5·75	33·06	6·35	40·32
3·96	15·68	4·56	20·79	5·16	26·63	5·76	33·18	6·36	40·45
3·97	15·76	4·57	20·88	5·17	26·73	5·77	33·29	6·37	40·58
3·98	15·84	4·58	20·98	5·18	26·83	5·78	33·41	6·38	40·70
3·99	15·92	4·59	21·07	5·19	26·94	5·79	33·52	6·39	40·83
4·00	16·00	4·60	21·16	5·20	27·04	5·80	33·64	6·40	40·96
4·01	16·08	4·61	21·25	5·21	27·14	5·81	33·76	6·41	41·09
4·02	16·16	4·62	21·34	5·22	27·25	5·82	33·87	6·42	41·22
4·03	16·24	4·63	21·44	5·23	27·35	5·83	33·99	6·43	41·34
4·04	16·32	4·64	21·53	5·24	27·46	5·84	34·11	6·44	41·47
4·05	16·40	4·65	21·62	5·25	27·56	5·85	34·22	6·45	41·60
4·06	16·48	4·66	21·72	5·26	27·67	5·86	34·34	6·46	41·73
4 07	16·56	4·67	21·81	5·27	27·77	5·87	34·46	6·47	41·86
4·08	16·65	4·68	21·90	5·28	27·88	5·88	34·57	6·48	41·99
4·09	16·73	4·69	22·00	5·29	27·98	5 89	34·69	6·49	42·12
4·10	16·81	4·70	22.09	5·30	28·09	5·90	34·81	6·50	42·25
4·11	16·89	4·71	22·18	5·31	28·20	5·91	34·93	6·51	42·38
4·12	16·97	4·72	22 28	5·32	28·30	5·92	35·05	6·52	42·51
4·13	17·06	4·73	22·37	5·33	28·41	5·93	35·16	6·53	42·64
4·14	17·14	4·74	22·47	5·34	28·52	5·94	35·28	6·54	42·77
4·15	17·22	4·75	22·56	5·35	28·62	5·95	35·40	6·55	42·90
4·16	17·31	4·76	22·66	5·36	28·73	5·96	35·52	6·56	43·03
4·17	17·39	4·77	22·75	5·37	28·84	5·97	35·64	6·57	43·16
4·18	17·47	4·78	22·85	5·38	28·94	5·98	35·76	6·58	43·30
4·19	17·56	4·79	22·94	5·39	29·05	5·99	35·88	6·59	43·43
4·20	17·64	4·80	23·04	5·40	29·16	6·00	36·00	6·60	43·56

TABLE IV. CONTINUED.

Squares of Numbers to two Decimal Places.

Root.	Square.	Root.	Square.	Root.	Square.	Root.	Square.
6·60	43·56	7·20	51·84	7·80	60·84	8·40	70·56
6·61	43·69	7·21	51·98	7·81	61·00	8·41	70·73
6·62	43·82	7·22	52·13	7·82	61·15	8·42	70·90
6·63	43·96	7·23	52·27	7·83	61·31	8·43	71·06
6·64	44·09	7·24	52·42	7·84	61·47	8·44	71·23
6·65	44·22	7·25	52·56	7·85	61·62	8·45	71·40
6·66	44·36	7·26	52·71	7·86	61·78	8·46	71·57
6·67	44·49	7·27	52·85	7·87	61·94	8·47	71·74
6·68	44·62	7·28	53·00	7·88	62·09	8·48	71·91
6·69	44·76	7·29	53·14	7·89	62·25	8·49	72·08
6·70	44·89	7·30	53·29	7·90	62·41	8·50	72·25
6·71	45·02	7·31	53·44	7·91	62·57	8·51	72·42
6·72	45·16	7·32	53·58	7·92	62·73	8·52	72·59
6·73	45·29	7·33	53·73	7·93	62·88	8·53	72·76
6·74	45·43	7·34	53·88	7·94	63·04	8·54	72·93
6·75	45·56	7·35	54·02	7·95	63·20	8·55	73·10
6·76	45·70	7·36	54·17	7·96	63·36	8·56	73·27
6·77	45·83	7·37	54·32	7·97	63·52	8·57	73·44
6·78	45·97	7·38	54·46	7·98	63·68	8·58	73·62
6·79	46·10	7·39	54·61	7.99	63·84	8·59	73·79
6·80	46·24	7.40	54·76	8·00	64·00	8·60	73·96
6·81	46·38	7·41	54·91	8·01	64·16	8·61	74·13
6·82	46·51	7·42	55·06	8·02	64·32	8·62	74·30
6·83	46·65	7·43	55·20	8·03	64·48	8·63	74·48
6·84	46·79	7·44	55·35	8·04	64·64	8·64	74·65
6·85	46·92	7·45	55·50	8·05	64·80	8·65	74·82
6·86	47·06	7·46	55·65	8·06	64·96	8·66	75·00
6·87	47·20	7·47	55·80	8·07	65·12	8·67	75·17
6·88	47·33	7·48	55·95	8·08	65·29	8·68	75·34
6·89	47·47	7·49	56·10	8·09	65·45	8·69	75·52
6·90	47·61	7·50	56·25	8·10	65·61	8·70	75·69
6·91	47·75	7·51	56·40	8·11	65·77	8·71	75·86
6·92	47·89	7·52	56·55	8·12	65·93	8·72	76·04
6·93	48·02	7·53	56·70	8·13	66·10	8·73	76·21
6·94	48·16	7·54	56·85	8·14	66·26	8·74	76·39
6·95	48·30	7·55	57·00	8·15	66·42	8·75	76·56
6·96	48·44	7·56	57·15	8·16	66·59	8·76	76·74
6·97	48·58	7·57	57·30	8·17	66·75	8·77	76·91
6·98	48·72	7·58	57·46	8·18	66·91	8·78	77·09
6·99	48·86	7·59	57·61	8·19	67·08	8·79	77·26
7·00	49·00	7·60	57·76	8·20	67·24	8·80	77·44
7·01	49·14	7·61	57·91	8·21	67·40	8·81	77·62
7·02	49·28	7·62	58·06	8·22	67·57	8·82	77·79
7·03	49.42	7·63	58·22	8·23	67·73	8·83	77·97
7·04	49·56	7·64	58·37	8·24	67·90	8·84	78·15
7·05	49·70	7·65	58·52	8·25	68·06	8·85	78·32
7·06	49·84	7·66	58·68	8·26	68·23	8·86	78·50
7·07	49·98	7·67	58·83	8·27	68·39	8·87	78·68
7·08	50·13	7·68	58·98	8·28	68·56	8·88	78·85
7·09	50·27	7·69	59·14	8·29	68·72	8·89	79·03
7·10	50·41	7·70	59·29	8·30	68·89	8·90	79·21
7·11	50·55	7·71	59·44	8·31	69·06	8·91	79·39
7·12	50·69	7·72	59·60	8·32	69·22	8·92	79·57
7·13	50·84	7·73	59·75	8·33	69·39	8·93	79·74
7·14	50·98	7·74	59·91	8·34	69·56	8·94	79·92
7·15	51·12	7·75	60·06	8·35	69·72	8·95	80·10
7·16	51·27	7·76	60·22	8·36	69·89	8·96	80·28
7·17	51·41	7·77	60·37	8·37	70·06	8·97	80·46
7·18	51·55	7·78	60·53	8·38	70·22	8·98	80·64
7·19	51·70	7·79	60·68	8·39	70·39	8·99	80·82
7·20	51·84	7·80	60·84	8·40	70·56	9·00	81·00

TABLE V.

	Arg. $\frac{p-u}{q-v}$ or $\frac{u}{v}$		Arg. $\frac{q-v}{p-u}$ or $\frac{v}{u}$			Arg. $\frac{p-u}{q-v}$ or $\frac{u}{v}$		Arg. $\frac{q-v}{p-u}$ or $\frac{v}{u}$	
Arg.	Arg. +	Arg. —	Arg. +	Arg. —	Arg.	Ar g +	Arg. —	Arg. +	Arg. —
·00	0°·0	180°·0	90°·0	90°·0	·50	26°·6	153°·4	63°·4	116°·6
·01	0 ·6	179 4	89 ·4	90 ·6	·51	27 ·0	153 ·0	63 ·0	117 ·0
·02	1 ·1	178 ·9	88 ·9	91 ·1	·52	27 ·5	152 ·5	62 ·5	117 ·5
·03	1 ·7	178 ·3	88 ·3	91 ·7	·53	27 ·9	152 ·1	62 ·1	117 ·9
·04	2 ·3	177 ·7	87 ·7	92 ·3	·54	28 ·4	151 ·6	61 ·6	118 ·4
·05	2 ·9	177 ·1	87 ·1	92 ·9	·55	28 ·8	151 ·2	61 ·2	118 ·8
·06	3 ·4	176 ·6	86 ·6	93 ·4	·56	29 ·2	150 ·8	60 ·8	119 ·2
·07	4 ·0	176 ·0	86 ·0	94 ·0	·57	29 ·7	150 ·3	60 ·3	119 ·7
·08	4 ·6	175 ·4	85 ·4	94 ·6	·58	30 ·1	149 ·9	59 ·9	120 ·1
·09	5 ·1	174 ·9	84 ·9	95 ·1	·59	30 5	149 ·5	59 ·5	120 ·5
·10	5 ·7	174 ·3	84 ·3	95 ·7	·60	31 ·0	149 ·0	59 ·0	121 ·0
·11	6 ·3	173 ·7	83 ·7	96 ·3	·61	31 ·4	148 ·6	58 ·6	121 ·4
·12	6 ·8	173 ·2	83 ·2	96 ·8	62	31 ·8	148 ·2	58 ·2	121 ·8
·13	7 ·4	172 ·6	82 ·6	97 ·4	·63	32 ·2	147 ·8	57 ·8	122 ·2
·14	8 ·0	172 ·0	82 ·0	98 ·0	·64	32 ·6	147 ·4	57 ·4	122 ·6
·15	8 ·5	171 ·5	81 ·5	98 ·5	·65	33 ·0	147 ·0	57 ·0	123 ·0
·16	9 ·1	170 ·9	80 ·9	99 ·1	·66	33 ·4	146 ·6	56 ·6	123 ·4
·17	9 ·6	170 ·4	80 ·4	99 ·6	·67	33 ·8	146 ·2	56 ·2	123 ·8
·18	10 ·2	169 ·8	79 ·8	100 ·2	·68	34 ·2	145 ·8	55 ·8	124 ·2
·19	10 ·8	169 ·2	79 ·2	100 ·8	·69	34 ·6	145 ·4	55 ·4	124 ·6
·20	11 ·3	168 ·7	78 ·7	101 ·3	·70	35 ·0	145 ·0	55 ·0	125 0
·21	11 ·9	168 ·1	78 ·1	101 ·9	·71	35 ·4	144 ·6	54 ·6	125 ·4
·22	12 ·4	167 ·6	77 ·6	102 ·4	·72	35 ·8	144 ·2	54 ·2	125 ·8
·23	13 ·0	167 ·0	77 ·0	103 ·0	·73	36 ·1	143 ·9	53 ·9	126 ·1
·24	13 ·5	166 ·5	76 ·5	103 ·5	·74	36 ·5	143 ·5	53 ·5	126 ·5
·25	14 ·0	166 ·0	76 ·0	104 ·0	·75	36 ·9	143 ·1	53 ·1	126 ·9
·26	14 ·6	165 ·4	75 ·4	104 ·6	·76	37 ·2	142 ·8	52 ·8	127 ·2
·27	15 ·1	164 ·9	74 ·9	105 ·1	·77	37 ·6	142 ·4	52 ·4	127 ·6
·28	15 ·6	164 ·4	74 ·4	105 ·6	·78	38 ·0	142 ·0	52 ·0	128 ·0
·29	16 ·2	163 ·8	73 ·8	106 ·2	·79	38 ·3	141 ·7	51 ·7	128 ·3
·30	16 ·7	163 ·3	73 ·3	106 ·7	·80	38 ·7	141 ·3	51 ·3	128 ·7
·31	17 ·2	162 ·8	72 ·8	107 ·2	.81	39 ·0	141 ·0	51 ·0	129 ·0
·32	17 ·7	162 ·3	72 ·3	107 ·7	.82	39 ·4	140 ·6	50 ·6	129 ·4
·33	18 ·3	161 ·7	71 ·7	108 ·3	.83	39 ·7	140 ·3	50 ·3	129 ·7
·34	18 ·8	161 ·2	71 ·2	108 ·8	.84	40 ·0	140 ·0	50 ·0	130 ·0
·35	19 ·3	160 ·7	70 ·7	109 ·3	·85	40 ·4	139 ·6	49 ·6	130 ·4
·36	19 ·8	160 ·2	70 ·2	109 ·8	·86	40 ·7	139 ·3	49 ·3	130 ·7
·37	20 ·3	159 ·7	69 ·7	110 ·3	·87	41 ·0	139 ·0	49 ·0	131 ·0
·38	20 ·8	159 ·2	69 ·2	110 ·8	·88	41 ·3	138 ·7	48 ·7	131 ·3
·39	21 ·3	158 ·7	68 ·7	111 ·3	·89	41 ·7	138 ·3	48 ·3	131 ·7
·40	21 ·8	158 ·2	68 ·2	111 ·8	·90	42 ·0	138 ·0	48 ·0	132 ·0
·41	22 ·3	157 ·7	67 ·7	112 ·3	·91	42 ·3	137 ·7	47 ·7	132 ·3
·42	22 ·8	157 ·2	67 ·2	112 ·8	·92	42 ·6	137 ·4	47 ·4	132 ·6
·43	23 ·3	156 ·7	66 ·7	113 ·3	·93	42 ·9	137 ·1	47 ·1	132 ·9
·44	23 ·7	156 ·3	66 ·3	113 ·7	·94	43 ·2	136 ·8	46 ·8	133 ·2
·45	24 ·2	155 ·8	65 ·8	114 ·2	·95	43 ·5	136 ·5	46 ·5	133 ·5
·46	24 ·7	155 ·3	65 ·3	114 ·7	·96	43 ·8	136 ·2	46 ·2	133 ·8
·47	25 ·2	154 ·8	64 ·8	115 ·2	·97	44 ·1	135 ·9	45 ·9	134 ·1
·48	25 ·6	154 ·4	64 ·4	115 ·6	·98	44 ·4	135 ·6	45 ·6	134 ·4
·49	26 ·1	153 ·9	63 ·9	116 ·1	·99	44 ·7	135 ·3	45 ·3	134 ·7
·50	26 ·6	153 ·4	63 ·4	116 ·6	1·00	45 ·0	135 ·0	45 ·0	135 ·0

TABLE VI.

Values of u and C *for Latitude of Philadelphia.*
Arg. *The Hour Angle* H.

Arg.	Arg.	*u.*	C.	Arg.	Arg.
180°	0°	0·00	7·68	90°	90°
179	1	0·13	7·68	89	91
178	2	0·27	7·67	88	92
177	3	0·40	7·67	87	93
176	4	0·54	7·66	86	94
175	5	0·67	7·65	85	95
174	6	0·80	7·63	84	96
173	7	0·93	7·62	83	97
172	8	1·07	7·60	82	98
171	9	1·20	7·58	81	99
170	10	1·33	7·56	80	100
169	11	1·46	7·54	79	101
168	12	1·60	7·51	78	102
167	13	1·73	7·48	77	103
166	14	1·86	7·45	76	104
165	15	1·99	7·41	75	105
164	16	2·12	7·38	74	106
163	17	2·24	7·34	73	107
162	18	2·37	7·30	72	108
161	19	2·50	7·26	71	109
160	20	2·63	7·21	70	110
159	21	2·75	7·17	69	111
158	22	2·88	7·12	68	112
157	23	3·00	7·07	67	113
156	24	3·12	7·01	66	114
155	25	3·24	6·96	65	115
154	26	3·36	6·90	64	116
153	27	3·48	6·84	63	117
152	28	3·60	6·78	62	118
151	29	3·72	6·71	61	119
150	30	3·84	6·65	60	120
149	31	3·95	6·58	59	121
148	32	4·07	6·51	58	122
147	33	4·18	6·44	57	123
146	34	4·29	6·36	56	124
145	35	4·40	6·29	55	125
144	36	4·51	6·21	54	126
143	37	4·62	6·13	53	127
142	38	4·73	6·05	52	128
141	39	4·83	5·97	51	129
140	40	4·93	5·88	50	130
139	41	5·04	5·79	49	131
138	42	5·14	5·70	48	132
137	43	5·24	5·61	47	133
136	44	5·33	5·52	46	134
135	45	5·43	5·43	45	135
		C.	*u.*		

TABLE VII.

Values of b and f for Latitude of Philadelphia.
Arg. *Sun's or Star's Declin.*

Arg.	*b.*	*f.*
0°	6·39	0·00
1	6·39	0·11
2	6·38	0·22
3	6·38	0·33
4	6·37	0·45
5	6·36	0·56
6	6·35	0·67
7	6·34	0·78
8	6·33	0·89
9	6·31	1·00
10	6·29	1·11
11	6·27	1·22
12	6·25	1·33
13	6·22	1·44
14	6·20	1·55
15	6·17	1·65
16	6·14	1·76
17	6·11	1·87
18	6·08	1·97
19	6·04	2·08
20	6·00	2·18
21	5·96	2·29
22	5·92	2·39
23	5·88	2·50
24	5·84	2·60
25	5·79	2·70
26	5·74	2·80
27	5·69	2·90
28	5·64	3·00
29	5·59	3·10
30	5·53	3·19

TABLE VIII.

Moon's reduced Semidiameter.
Arg. *Moon's Horizontal Parallax.*

Arg.	Semidiam.
53′	2·7324
54	2·7323
55	2·7321
56	2·7320
57	2·7319
58	2·7318
59	2·7316
60	2·7315
61	2·7314
62	2·7313

TABLE IX.

Correction of r, the Sun's Reduced Semidiameter.
Arguments, $(f + g)$ *at the Top, and Sun's Semidiameter at the Side.*

	1	2	3	4	5	6	7	8	9	10
15′ 46″	·0046	·0092	·0138	·0184	·0230	·0276	·0322	·0368	·0414	·0460
48	46	92	38	84	30	76	23	69	15	61
15 50	·0046	·0092	·0139	·0185	·0231	·0277	·0323	·0369	·0416	·0462
52	46	93	39	85	31	78	24	70	16	63
54	46	93	39	85	32	78	25	71	17	64
56	46	93	39	86	32	79	25	72	18	65
58	47	93	40	86	33	79	26	72	19	66
16 0	·0047	·0093	·0140	·0187	·0233	·0280	·0327	·0373	·0420	·0467
2	47	94	40	87	34	81	27	74	21	68
4	47	94	41	87	34	81	28	75	22	68
6	47	94	41	88	35	82	29	76	23	69
8	47	94	41	88	35	82	29	76	23	70
16 10	·0047	·0094	·0141	·0189	·0236	·0283	·0330	·0377	·0424	·0471
12	47	94	42	89	36	83	31	78	25	72
14	47	95	42	89	37	84	31	79	26	73
16	47	95	42	90	37	85	32	80	27	74
18	48	95	43	90	38	85	33	80	28	75